Journal of Engineering Physics and Thermophysics, Vol. 74, No. 5, 2001

DEFORMATIONS OF ROCKS IN PERIODIC REGIMES OF FILTRATION

M. N. Ovchinnikov, A. Yu. Zavidonov, and S. P. Evtushenko

UDC 531.4+622.276

The authors present results of experimental investigations of deformations of rocks that occur in nonstationary filtration of a liquid in thin deep-lying cracked-porous strata. Equations for description of lateral displacements and deformations that satisfactorily describe the data of the experiments are obtained.

Introduction. The problem of monitoring of filtration flows is pressing for the petroleum industry at present. Different geophysical and hydrodynamic methods are employed for its solution. Among other methods, of interest are those that are based on investigating the deformations of rocks occurring in the process of filtration (see, for example, [13]). A certain advantage of such methods over hydrodynamic ones is that sensors, recording displacements or deformations, can be located at any points of the earth's surface and not be "tied" to wells that receive hydrodynamic disturbances of pressure, which considerably extends the capabilities of monitoring flows of liquids in strata. In this work, we consider the issues of description of deformations of rocks occurring in periodic regimes of filtration of liquids.

Description of the Experiment. Experimental investigations of deformations of rocks occurring in the process of filtration were carried out by S. P. Evtushenko on a section of the Romashkino deposit with a cracked-porous collector lying at a depth of about 700 m with a thickness of a stratum of ~10–15 m and a permeability of $\sim 10^{-12}$ m^2. It was assumed that in cracked-porous strata, strong deformations would occur due to the fact that the compressibility of such media is much higher than that of merely porous media. The investigations carried out by the method of filtration harmonic pressure waves showed that the piezoconductivity in interwell intervals varied within 0.5–1.5 m^2/sec; the pressure drops in the regimes injection–outflow attained 6 MPa.

Observation of the deformations was carried out by measuring the relative displacement of two points located on one vertical near the earth's surface. For this purpose, one drilled a 7.5-m-deep pit at whose bottom the lower end of a 5-m-long rod vertically installed in the pit was cemented; the lower plate of a sensor was fixed on the free end of the rod. The upper plate of the capacitance sensor with a circuit was fixed on a cement base at a depth of 2.5 m. Thus, the working base of the device was 5 m. The frequency of the resonance circuit changed with change in the relative distance between the plates of the sensors, which was recorded by a frequency meter. Two measuring devices were arranged at distances of 75 m (device 1) and 150 m (device 2) in different directions from a well that prescribed pressure disturbances in the stratum.

As a result of prescribing variable discharges on this well in regimes of the form injection–outflow and injection–downtime with periods of 57,600, 86,400, 129,600, and 172,800 sec, the pressure in the stratum changed, which in principle must lead to periodic deformations of rocks and to the displacements of the earth's surface. And indeed, such deformations were observed, and very clearly for relatively long (more than 10^5 sec) periods of oscillations of the discharges of the disturbing well since the long periods of pressure

Kazan State University, Kazan, Russia; email: Marat.Ovchinnikov@ksu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 5, pp. 13–16, September–October, 2001. Original article submitted February 6, 2001.

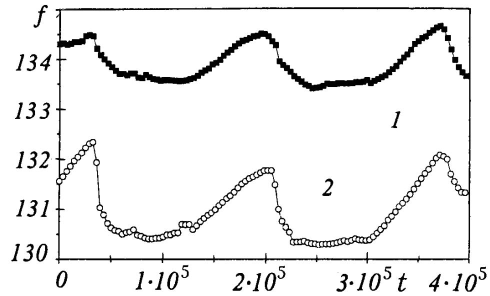


Fig. 1. Recording (carried out by two devices) of signals induced by deformations of the stratum in the process of filtration in the case of successive recurrence of oscillations of the discharge of a disturbing well (curves 1 and 2). *f*, kHz; *t*, sec.

oscillations correspond to large amplitudes of change of the pressures in the stratum. The oscillations of the discharges with each individual period were realized in the form of series of 3–6 successive repetitions of discharge oscillations with this period.

Thus, Fig. 1 presents the dynamics of readings of the sensors in frequencies when pressure periods of the same duration are assigned on the disturbing well. It is seen that all curves repeat their maxima and minima, which indicates an identical, at least in terms of quality, reaction of the system under study to nonstationary disturbances. So, in what manner can we describe deformations occurring in rocks?

Lateral Deformations of Rocks Caused by Filtration. Let us consider a fluid-saturated stratum of thickness h that lies at a depth H. We assume that the stratum is drilled by a single well through which the motion of the fluid is executed. Then we will assume everywhere that the condition $h \ll H$ is fulfilled, which is usually observed in practice. Subsequently, in order to determine deformations in the stratum, we will follow the procedure of [2–4].

By virtue of the axial symmetry of the problem we adopt a cylindrical coordinate system; the x axis coincides with the axis of the well, while the plane $z = H$ is perpendicular to the force of gravity and is a free surface. The plane $z = 0$ coincides with the middle plane of the stratum. The stratum is considered to be homogeneous, and the motion of the liquid is considered to be axisymmetric. We assume that the rock is characterized by the averaged Poisson coefficient σ and Young modulus E, while the characteristic of the stratum is the rigidity of lateral deformations E_{str}/h.

Solution of the problem of the displacements of a free surface and deformations is reduced to finding the displacement vector $\mathbf{u}$ of the medium. The quantity $\mathbf{u}$ satisfies an equation of the following form:

$$\text{grad}\,(\text{div}\,\mathbf{u}) - \frac{1-2\sigma}{2\,(1-\sigma)}\,\text{rot}\,(\text{rot}\,\mathbf{u}) = 0\,. \tag{1}$$

Boundary conditions for this equation are written in the form [2]

$$\sigma_z = \tau_{rz} = 0 \quad (z = H)\,, \quad \Delta\sigma_z = \Delta\tau_{rz} = \Delta u_r = 0\,, \quad \Delta u_z = (\sigma_z + p\,(r, t))\,h/E_{\text{str}} \quad (z = h/2 \approx 0)\,, \tag{2}$$

where Δ denotes the difference (jump) of the corresponding quantity at the boundary rock–stratum and $p(r, t)$ is the pressure in the stratum, which is determined from the filtration equation. We note that the condition $z = h/2 \approx 0$ for Δu_z is appropriate in the case where $h \ll H$.

All the quantities in (2) can be calculated accurately by using the Papkovich–Naiber representation for the components of the stress tensor and the vector $\mathbf{u}$ in terms of the harmonic functions Φ and φ in the form of Hankel integrals [3, 4]:

$$\sigma_z = 2\,(1-\sigma)\,\frac{\partial \Phi}{\partial z} - z\,\frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial^2 \varphi}{\partial z^2}\,, \quad \tau_{rz} = \frac{\partial}{\partial r}\left[(1-2\sigma)\,\Phi - z\,\frac{\partial \Phi}{\partial z} - \frac{\partial \varphi}{\partial z}\right],$$

$$u_z = \frac{1+\sigma}{E}\left[(3-4\sigma)\,\Phi - z\,\frac{\partial \Phi}{\partial z} - \frac{\partial \varphi}{\partial z}\right], \quad u_r = -\,\frac{1+\sigma}{E}\,\frac{\partial}{\partial r}\,[\varphi + z\Phi]\,. \tag{3}$$

For the pressure $p(r, t)$ the integral representation of Hankel will be written as

$$p\,(r, t) = \int_0^\infty P\,(\xi, t)\,J_0\,(\xi r)\,\xi d\xi\,. \tag{4}$$

Now we can obtain the formula for the displacements of the free surface of a rock $u_z(z = 11)$ in the form

$$u_z\,(r, t, z = H) = \frac{4\,(1-\sigma^2)\,H}{E}\int_0^\infty d\xi\xi J_0\,(\xi r)\,\exp\,(-\,H\xi)\,\frac{(1 + H\xi)\,P\,(\xi, t)}{a + H\xi f\,(\xi)}\,, \tag{5}$$

where

$$a = \frac{4\,(1-\sigma^2)\,HE_{\text{str}}}{Eh}\,; \quad f\,(\xi) = 1 - (1 + 2H\xi + 2H^2\xi^2)\,\exp\,(-\,2H\xi)\,,$$

and for the deformations it will be written as

$$u_{zz}\,(r, t, z = H) = -\,\frac{16\sigma\,(1+\sigma)\,H^2}{E}\int_0^\infty d\xi\,\frac{\xi^3 J_0\,(\xi r)\,\exp\,(-\,H\xi)\,P\,(\xi, t)}{a + \xi H f\,(\xi)}\,. \tag{6}$$

Next a certain form of the function $P(\xi, 1)$ that corresponds to the specific problem must be substituted into (5) or (6). For this purpose we should find the pressure $p(r, t)$, taking into account the deformations occurring in the process of filtration.

With account for the compressibility of the stratum and disregarding the displacements of rocks in the stratum's plane, we can modify the classical equation of filtration in the form [5]

$$c\,\frac{\partial p}{\partial t} + d\,\frac{\partial \sigma_z}{\partial t} = \frac{k}{\mu}\,\nabla^2 p\,, \tag{7}$$

where the parameters c and d are determined in terms of the elastic properties of the stratum and the fluid.

The form of the solution of (7) depends on the method of assignment of a load on the stratum [5]. Thus, for the case of a constant vertical load produced by the weight of the rock massif from the continuity condition on an impermeable roof and bottom of the stratum it follows that $\partial p/\partial t = \partial \sigma_z/\partial t$ and (7) is reduced to the Fourier equation for the filtration pressure

$$\frac{\partial p}{\partial t} = \chi \nabla^2 p\,, \tag{8}$$

with the piezoconductivity of the stratum $\chi = k/[\mu(c + d)]$.

For cracked-porous media, following the Barenblatt model [6], we can write the filtration equation in the case of this method of assignment of the load in the form

$$\frac{\partial p}{\partial t} = \chi \nabla^2 \left(p + \tau \frac{\partial p}{\partial t} \right), \tag{9}$$

where τ is the dimension constant of time characterizing the exchange of a liquid between blocks and cracks.

For the latter equation the function $P(\xi, t)$ has the form

$$P(\xi, t) = \frac{\chi}{2\pi\varepsilon} \frac{1}{(1 + \chi\tau\xi^2)^2} \int_0^t dt' q(t - t') \exp\left(- \frac{t'\chi\xi^2}{1 + \chi\tau\xi^2} \right), \tag{10}$$

and in assignment of the discharge in the disturbing well in the form of the periodic $q(t) = q_0 \sin \omega t$ function with a period $T = 2\pi/\omega$ the function $P(\xi, t)$ (for the times $t >> T$) will be written as

$$P(\xi, t) = \frac{q_0}{2\pi\varepsilon} \frac{\chi}{(1 + \chi\tau\xi^2)^2} \left[\frac{b}{\omega^2 + b^2} \sin \omega t - \frac{\omega \cos \omega t}{\omega^2 + b^2} + \frac{\omega}{\omega^2 + b^2} \exp(-bt) \right], \quad b = \frac{\chi\xi^2}{1 + \chi\tau\xi^2}. \tag{11}$$

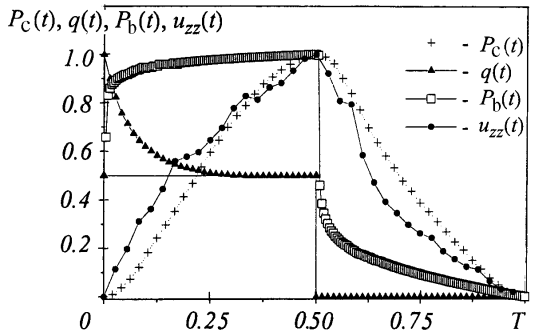
According to another method of assignment of the load, the changes in the threshold pressure in the stratum are related to the displacements of the rock massif. In this case, (8) is reduced to the integro-differential equation

$$\frac{\partial}{\partial t} \left[p - \alpha \int_0^\infty \frac{\xi^2 F(\xi, t)}{\xi + \lambda} J_0(\xi r)\, d\xi \right] = \chi \nabla^2 p. \tag{12}$$

Here $F(\xi, t) = \int_0^\infty y p(y, t) J_0(\xi y) dy$ and $\lambda = a/H$ and α is the dimensionless coefficient determined by the compressibilities of the rocks and the fluid.

We note that the integral in the brackets of expression (12) usually has no substantial effect [4]; therefore, in calculations we can use formulas (8)–(11).

Interpretation of the Experiment. In carrying out hydrodynamic experiments, periodic oscillations of the discharges and pressures were produced by successively recurrent openings and closures of the disturbing well in certain intervals of time. We note that, under actual conditions, the shape of the pulses of the discharge can differ from a sinusoidal one and we can write it in the form of a superposition of functions and harmonics of the corresponding Fourier expansion that approximate the discharge. Thus, for the situation in question, the shape of discharge pulses for an individual period has the form of triangles (Fig. 2); the squares show the corresponding pressures at the bottom of the disturbing well. Here, during the first half of the period, there is a nonzero discharge and a pressure increase is observed, while in the second half of the period the discharge is zero and the pressure decreases. As the harmonic propagates in the medium, pressures of higher order decay more rapidly than pressures of lower order; therefore, the shape of pressure pulses will change (the initial rectangles "degenerate" into sinusoids), which is shown in Fig. 2 as crosses for pressures that are measured in the control well located at a distance of 98 m from the disturbing one. The form of the occurring deformations recorded by the device at a distance of 150 m from the injecting well is shown as circles (all the above quantities are dimensionless and are normalized to their maximum values).



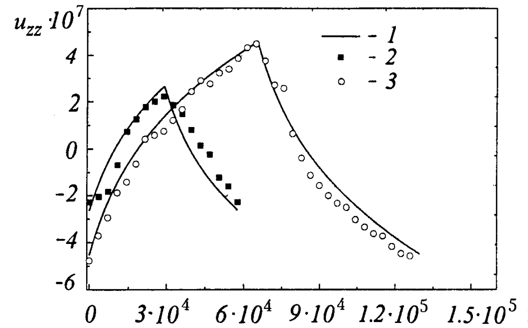


Fig. 2. Time changes of the discharge $q(t)$ and the pressure $P_b(t)$ in a disturbing well, of the pressure $P_c(t)$ in a control well, and of the deformation $u_{zz}(t)$ that are recorded by the device.

Fig. 3. Comparison of the calculated (1) and experimental values of the deformations for discharge oscillations with periods of 57,600 sec (2) and 129,600 sec (3). *t*, sec.

The deformations experimentally measured for the periods of oscillations of the discharge on the disturbing well are shown in Fig. 3. The measurements were carried out at a point at a distance of 150 m from the well. The lines show results of calculating from formula (6) using (8) and (10) with account for the actual shape of pressure pulses that are shown in Fig. 2 as the triangles. In obtaining these solutions, we employed the self-similar solution for pressure; this solution disregards the finite radius of the well, which turns out to be justified in the situation in question. We also disregarded the effects that are caused by the pressure redistribution between blocks and cracks, taking into account that the condition $\tau << T$ was fulfilled (see also [4]). We took the following values of the parameters: Poisson coefficient $\sigma = 0.23$, Young modulus of the stratum $E_{str} = 0.81\cdot10^4$ MPa (it was used as the fitting parameter), $E = 4\cdot10^4$ MPa, depth of occurrence $H = 690$ m, and stratum thickness $h = 11$ m; according to the results of the hydrodynamic experiments, the calculated value of the piezoconductivity was taken to be equal to $\chi = 0.65$ m^2/sec, $q_0/(2\pi\varepsilon) = 1.70$ MPa.

Quite a satisfactory agreement between theoretical curves and experimental points is obvious.

Conclusions. The theoretical approaches given in this work are suitable for description of lateral deformations occurring in nonstationary filtration in strata. These approaches can be used in investigations of both porous media and cracked-porous media with different methods of assignment of a load on a stratum. Within the framework of solution of applied problems, we can speak of the possibility of monitoring filtration flows through creation of a surface network of sensors that record the displacements and deformations of rocks.

NOTATION

h, stratum thickness, m; *H*, depth of occurrence of the stratum, m; *r*, distance from the well axis, m; *z*, coordinate in a cylindrical coordinate system, m; *t*, time, sec; *f*, frequency of the resonant circuit of the device, kHz; E_{str}, Young modulus of the stratum skeleton, Pa; *E*, Young modulus of the rocks, Pa; σ, Poisson coefficient of the rocks; **u**, u_z and u_r, vector of lateral displacements and its components, m; u_{zz}, lateral deformations; σ_z and τ_{rz}, components of the stress tensor, Pa; $p(r, t)$, pressure, Pa; *k*, permeability of the stratum, m^2; μ, viscosity of the fluid, Pa·sec; χ, piezoconductivity of the stratum, m^2/sec; ε, hydraulic conductivity of the stratum, m^3/(Pa·sec); *q* and q_0, discharge and amplitude of oscillations of the discharge of the well, m^3/sec; *T*, period of oscillations of the discharge of the well, sec; ω, cyclic frequency of oscillations of the discharge, rad/sec; τ, dimension constant of time in (9), sec; $P_b(t)$, pressure at the bottom of the disturbing well, Pa; $P_c(t)$, pressure in the control well, Pa; J_0, Bessel function; $P(\xi, t)$, transform of pressure in the Hankel representation, N; ξ, dimension variable of the inverse length in (4)–(6) and (10)–(12), m^{-1}; a, dimensionless parameter in (5) and (6); b, parameter in (10), sec^{-1}; c and d, elastic parameters in (7), Pa^{-1}; λ, parameter in (12), m^{-1}.

REFERENCES

1. I. D. Umrikhin, S. N. Buzinov, O. V. Kurenkov, V. M. Entov, and T. A. Malakhova, *Method for Determining the Coefficient of Hydropiezoconductivity of a Stratum*, Inventors's Certificate No. 697704 SSSR, Byull. Izobret., No. 4 (1979).
2. W. Novacki, in: *Teoria Sprężystości* [Russian translation], Moscow (1975), pp. 184–186.
3. V. M. Entov and T. A. Malakhova, *Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela*, No. 6, 53–65 (1974).
4. R. N. Diyashev, A. V. Kosterin, and É. V. Skvortsov, in: *Filtration of a Liquid in Deformable Oil Strata* [in Russian], Kazan (1999), pp. 162–187.
5. V. N. Nikolaevskii, in: *Geomechanics and Fluid Dynamics* [in Russian], Moscow (1984), pp. 112–116.
6. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, in: *Motion of Liquids and Gases in Natural Strata* [in Russian], Moscow (1984), pp. 104–114.